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Massive vectors and loop observables: the $g - 2$ case

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Abstract

We discuss the use of massive vectors for the interpretation of some recent experimental anomalies, with special attention to the muon $g - 2$. We restrict our discussion to the case where the massive vector is embedded into a spontaneously broken gauge symmetry, so that the predictions are not affected by the choice of an arbitrary energy cut-off. Extended gauge symmetries, however, typically impose strong constraints on the mass of the new vector boson and for the muon $g - 2$ they basically rule out, barring the case of abelian gauge extensions, the explanation of the discrepancy in terms of a single vector extension of the standard model. We finally comment on the use of massive vectors for B-meson decay and di-photon anomalies.

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Contents

1 Introduction

In the recent years there has been quite a lot of interest for the emergence of a few 3-4σ experimental anomalies in particle physics. Among those, the most relevant are the longstanding one of the anomalous magnetic moment of the muon, $(g-2)_\mu$, [1] (see Ref. [2] for a review) and a collection of anomalies in semileptonic B-meson decays [3–5]. More recently, ATLAS [6, 7] and CMS [8–10] reported a hint of a di-photon resonance with mass in the vicinity of 750 GeV in the first LHC data collected at 13 TeV collision energies.[1] None of them is conclusive at the moment, and require further scrutiny both from the experimental and the theoretical point of view; it is nevertheless tantalizing to try to interpret them within new physics frameworks beyond the standard model (SM). This has triggered a large amount of works, ranging from full-fledged theoretical constructions, like for example supersymmetry, up to simplified 1-particle extensions of the SM. In the latter case, one simply adds a new irreducible representation (irrep) on top of the SM field content, with spin quantum number 0, 1/2, 1, etc. While the case of a new scalar or fermion irrep is conceptually straightforward, being the SM extension automatically renormalizable and well-behaved in the ultraviolet (UV), the one of a generic Lorentz vector is less obvious and will be the subject of the present paper.

The two main challenges that one faces when extending the SM with a vector irrep are the following: *i*) depending on the UV completion, the theory might not be renormalizable, thus reducing the degree of predictivity for the observables whose anomaly one is willing to explain and *ii*) regardless of the renormalizability issue, the 1-particle extensions hypothesis is possibly violated in explicit constructions, which require several new particles at the same energy scale.

Concerning the first point, massive vectors typically arise either as composite states resulting from a new strongly-coupled dynamics (for example the ρ meson in QCD) or as extra gauge bosons associated with a spontaneously broken gauge extension of the SM. The difference between these two possibilities is substantial, the most dramatic being renormalizability. Though there is nothing wrong in contemplating a non-renormalizable theory within an effective field theory (EFT) approach, we will focus on UV-complete, weakly-coupled models which provide a more predictive framework for dealing with precision loop observables. As a prototypical example we will mainly discuss the $(g-2)_\mu$, while commenting *en passant* on other anomalies.

After a brief review of the $(g-2)_\mu$ discrepancy in Sect. 2, we discuss in Sect. 3 the most general $d \leq 4$ Lagrangian of a massive vector coupled to the SM, and show the divergence structure of the one-loop diagrams. In the particular case at hand, we will see that the culprit of the non-renormalizability resides in the triple vector boson vertex which has to be properly modified in order for the theory to be renormalizable. In Sect. 4 we classify all possible SM gauge quantum numbers of the new vector, hereafter denoted by X, coupling to a muon and to another SM fermion (a general classification of the X gauge quantum numbers such that it couples to SM fields at the renormalizable level is provided in Appendix A). Next, by assuming that X is a gauge boson of an extended SM gauge group, we compute for each case the finite contribution to the $(g-2)_\mu$ and estimate the required mass scale, M_X, in order to explain the discrepancy. Remarkably, after providing a minimal gauge embedding for each case, we find that the UV theory imposes strong direct and indirect constraint (e.g. from proton decay or flavor violating processes), such that most of the simplified 1-particle extended models cannot provide an explanation of the $g-2$ discrepancy in the full renormalizable setup. The only exception to

[1] New 2016 LHC data at 13 TeV have not confirmed the excess [11, 12].

this rule is given by abelian gauge extensions, like e.g. the case of a light dark photon or dark Z. Furthermore, another aspect emerging from the full analysis is that extra states required by the consistency of the gauge symmetry breaking pattern cannot be arbitrarily decoupled from X, thus typically violating the 1-particle extension hypothesis. We finally conclude in Sect. 5 by summarizing our findings and comment on the use of massive vectors for the B-meson decay and di-photon anomalies.

2 Review of the $(g-2)_\mu$ discrepancy

Known respectively with 12 and 9 digits, the anomalous magnetic moments of the electron and the muon are among the best measured quantities in physics. While the former is used to fix the value of the fine structure constant $\alpha_{\rm em}$, the latter constitutes a good observable where to look for new physics.

The world average of the measured $a_\mu \equiv (g_\mu - 2)/2$, dominated by the result obtained by E821 at Brookhaven [1], is given by [2]

$$a_\mu^{\rm exp} = 116592080(63) \cdot 10^{-11} \,. \tag{1}$$

In the SM a_μ arises at one loop and, due to the great precision of this measurement, higher order corrections must be taken into account. The SM contribution can be divided into three categories: *i*) QED contributions, consisting of loops involving only leptons and photons, *ii*) electroweak contributions, involving leptons, W, Z and Higgs bosons and *iii*) hadronic contributions, with hadronic resonances circulating in the loops. The QED contribution has been calculated up to five loops and the electroweak one up to two loops, which is enough for the current experimental precision. On the other hand, the largest error on the theoretical determination comes from the hadronic contributions: in the light-by-light scattering amplitude some theoretical input is needed in order to perform the calculation, while in the vacuum polarization diagrams some dispersion relations are extracted from experiments, either from e^+e^- scattering or from τ decay. Depending on these different inputs, different results are obtained for the theoretical prediction. We choose as a reference value for the SM determination the one contained in the review [2], while a list of other predictions can be found for instance in Ref. [13]:

$$a_\mu^{\rm SM} = 116591790(65) \cdot 10^{-11} \,. \tag{2}$$

If we now compare this with the measured value, we get a difference of $\Delta a_\mu = 290(90) \cdot 10^{-11}$ which corresponds to a discrepancy with 3.1σ significance. By choosing different theoretical predictions one obtains discrepancies which range from 2 to 4 σ. New, independent measurements are expected in the next few years by two collaborations, E989 at Fermilab [14] and E34 at JPARC [15], and therefore the existence of a $(g-2)_\mu$ anomaly will soon be confirmed or disproved; for the moment, we stick to the available experimental result.

Even if this is not enough to claim a discovery, this discrepancy deserves a detailed analysis. Basically, it can arise for two different reasons: either *i*) the SM prediction is not accurate, or *ii*) there is some physics beyond the SM contributing to the $(g-2)_\mu$.

Due to the difficulties in calculating the hadronic contributions, one could think that *i*) is the favourite explanation. However, if one fixes the hadronic contribution in order to agree with $a_\mu^{\rm exp}$, deviations in the electroweak precision observables are obtained. In particular, the Higgs mass prediction is modified and, in order to be compatible with the measured value, large modifications of the hadronic contribution at energies lower than 1 GeV would be required, while this is precisely the energy region where the experimental measurement is solid [16]. Therefore, this explanation seems to be disfavoured.

According to case *ii*), the discrepancy Δa_μ could be due to the presence of new physics beyond the SM. Indeed, in models beyond the SM involving new particles' couplings to muons, like for example supersymmetric models, a positive (and large) contribution to the $(g-2)_\mu$ can be achieved quite easily. Two approaches are therefore possible: either one takes a model, conceived to solve another problem, and verifies whether it can also explain this discrepancy, or tries to classify, in a more model-independent way, which are the new particles that can contribute to the $(g-2)_\mu$. This second approach is the one adopted e.g. in Ref. [17], where minimal extensions of the SM with a single scalar or fermion irrep were considered (see also Refs. [18–21] for other analysis with a similar formulation). In the present paper, we follow the same idea and complete the classification by adding one massive vector to the SM field content.

3 EFT approach to the $(g-2)_\mu$

A possible approach to the $(g-2)_\mu$ consists in adding to the SM field content a new Lorentz vector, X^μ, without specifying the full UV completion of the theory. In general, the theory is non-renormalizable and one expects loop observables to be divergent. In this section, we discuss the $d \leq 4$ operators that can appear in the Lagrangian of a massive vector coupled to the SM and analyze the divergence structure of the diagrams relevant for the $g-2$.

3.1 Lagrangian of a massive vector

Before performing the actual $(g-2)_\mu$ calculation, we discuss the Lagrangian of the new vector boson, which is assumed to transform under a complex irrep of the SM gauge group. As already mentioned, we will not assume that its mass originates from a spontaneously broken gauge symmetry. The canonical kinetic and mass terms of X^μ read[2]

$$\mathcal{L}_X^{\text{free}} = -\partial_\mu X_\nu^\dagger \partial^\mu X^\nu + \partial_\mu X_\nu^\dagger \partial^\nu X^\mu + M_X^2 X_\mu^\dagger X^\mu \,, \tag{4}$$

with propagator

$$i\Delta^{\mu\nu}(k) = \frac{i}{k^2 - M_X^2}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{M_X^2}\right) , \tag{5}$$

which is the same as the unitary gauge propagator of a massive gauge boson.

We are interested in working out the interaction term of X^μ with the photon field A^μ, which in turn contributes to the $g-2$. The so-called minimal coupling to electromagnetism is generated by simply replacing ordinary derivatives in Eq. (4) by covariant derivatives:

$$\partial_\mu X_\nu \to D_\mu X_\nu = (\partial_\mu - ieQ_X A_\mu) X_\nu, \tag{6}$$

[2]Note that by Lorentz and gauge invariance the most general Lagrangian quadratic in X^μ is

$$\mathcal{L}_X^{\text{free}} = -\partial_\mu X_\nu^\dagger \partial^\mu X^\nu + \beta\, \partial_\mu X_\nu^\dagger \partial^\nu X^\mu + M_X^2 X_\mu^\dagger X^\mu \,, \tag{3}$$

where β is a free parameter. It can be shown [22] that for $\beta = 1$ the above Lagrangian describes the free propagation of a massive spin 1 particle. For $\beta \neq 1$ a scalar degree of freedom is included as well.

where Q_X is the electric charge of X in units of the proton charge e. This is enough to make the Lagrangian of Eq. (3) invariant upon local gauge transformations

$$X^\mu \to e^{ieQ_X\alpha(x)} X^\mu ; \qquad A_\mu \to A_\mu + \partial_\mu \alpha(x) \,, \tag{7}$$

where $\alpha(x)$ is the local parameter of the transformation. The resulting coupling of the vector field X is

$$\mathcal{L}_X^{\rm em} = J_\mu^{\rm em} A^\mu - e^2 Q_X^2 \left(A_\mu A^\mu X_\nu^\dagger X^\nu - A_\mu A^\nu X_\nu^\dagger X^\mu \right) \,, \tag{8}$$

where

$$J_\mu^{\rm em} = ieQ_X \left[\left(\partial_\mu X_\nu^\dagger - \partial_\nu X_\mu^\dagger \right) X^\nu - \left(\partial_\mu X_\nu - \partial_\nu X_\mu \right) X^{\dagger\nu} \right] \,, \tag{9}$$

is the conserved current of the free theory.

On the other hand, it is easy to see that there exist extra gauge invariant terms not related to the minimal coupling. A complete classification of SM gauge invariant $d \leq 4$ operators involving X and SM fields is given in Appendix A, and the most general EFT should contain them all.

3.2 Divergence structure of one-loop diagrams

The EFT described in the previous subsection is non-renormalizable because X^μ is not a gauge boson. This can be proved on general grounds. However, it is interesting to see how non-renormalizability manifests itself in the case of the $g-2$, and to study its relationship with the minimal coupling. To this purpose we extend the minimally coupled theory by adding a gauge invariant term proportional to (see also Ref. [23])

$$\left(X_\mu X_\nu^\dagger - X_\nu X_\mu^\dagger \right) \partial^\mu A^\nu \,. \tag{10}$$

By including also the interaction terms with the muon field μ and a generic SM fermion f, the effective Lagrangian relevant for the $(g-2)_\mu$ calculation is

$$\begin{aligned} \mathcal{L}_{\rm int}^{g-2} &= \overline{\mu} \left(g_V \gamma_\alpha + g_A \gamma_\alpha \gamma_5 \right) f X^\alpha + \text{h.c.} \\ &+ ieQ_X \left[\left(\partial_\mu X_\nu^\dagger - \partial_\nu X_\mu^\dagger \right) A^\mu X^\nu - \left(\partial_\mu X_\nu - \partial_\nu X_\mu \right) A^\mu X^{\dagger\nu} + k_Q \left(X_\mu X_\nu^\dagger - X_\nu X_\mu^\dagger \right) \partial^\mu A^\nu \right] \\ &+ ieQ_f \overline{f} \gamma_\mu f A^\mu \,, \end{aligned} \tag{11}$$

where $g_{V,A}$ are vector and axial couplings, Q_f is the electromagnetic (EM) charge of f and k_Q is a free parameter.

The two diagrams contributing to the $(g-2)_\mu$ at the one-loop level are displayed in Fig. 1. The degree of superficial divergence of diagrams (a) and (b) is respectively 4 and 2. However, denoting by Λ the cut-off regulator, an explicit calculation shows that

- The contribution to the $(g-2)_\mu$ from diagram (a) is only logarithmically divergent, since the Λ^4 term vanishes when the virtuality of the external photon is set to zero, while the Λ^2 term goes into the renormalization of the electric charge.
- Diagram (b) is finite.

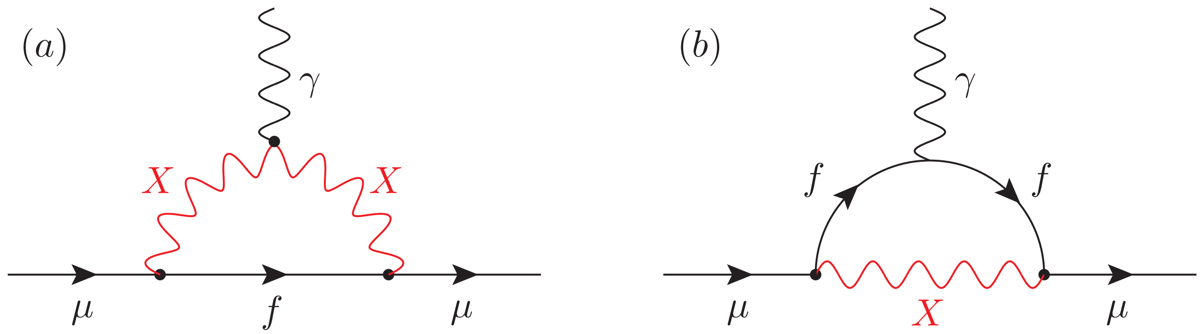

Figure 1: One-loop diagrams contributing to the $(g-2)_\mu$. Red wiggled lines stand for the massive vector X, while the blobs in the vertices denote the interactions of X with the SM fields defined in Eq. (11).

The reduction of the degree of divergence for the 3-point function is a simple consequence of the Ward identity, which connects the $\mu\mu\gamma$ vertex $\Gamma^\alpha(p,q)$ to the derivative of the muon self-energy $\Sigma(p)$ in the soft-photon limit $q \to 0$ via the relation

$$\Gamma^\alpha(p,p) = \frac{d\Sigma(p)}{dp_\alpha}\,. \tag{12}$$

To see this, let us Taylor expand the muon self-energy in powers of $\not{p} - m_\mu$

$$\Sigma(p) = A + B(\not{p} - m_\mu) + \Sigma_c(p)(\not{p} - m_\mu)\,. \tag{13}$$

Since $\Sigma(p)$ is linearly divergent, the first two coefficients A and B are respectively linearly and logarithmically divergent (indeed every derivative with respect to p lowers the degree of divergence by one unit). This implies that $d\Sigma(p)/dp_\alpha$, and hence $\Gamma^\alpha(p,p)$ because of Eq. (12), can be at most logarithmically divergent.

By employing the Lagrangian in Eq. (11) we find the following contribution to the divergent part of the $(g-2)_\mu$:

$$\Delta a_\mu^{\text{div.}} = \frac{Q_X m_\mu^2}{8\pi^2 M_X^2}(k_Q - 1)\left[\left(g_V^2 + g_A^2\right) - \frac{m_f}{m_\mu}\left(g_V^2 - g_A^2\right)\right]\log\frac{\Lambda^2}{M_X^2}\,. \tag{14}$$

This result shows that the logarithmic divergence disappears in the limit $k_Q \to 1$. On the other hand, the divergence persists in the minimally coupled theory ($k_Q = 0$). Also note that for $k_Q = 1$ and $Q_X = 1$, the second line of Eq. (11) reproduces the SM triple gauge vertex $WW^\dagger A$, with the identification $X^\mu = W^{+\mu}$. We hence conclude that the choice $k_Q = 1$ is a necessary condition for renormalizability. Moreover, possible extra gauge invariant terms in Eq. (11) do not arise in renormalizable theories (cf. the discussion in Appendix A).

Even though one could estimate the contribution of the massive vector to the $(g-2)_\mu$ by setting Λ to the value of the cut-off of the EFT, this requires the specification of a new energy scale (e.g. the scale of compositeness in strongly-coupled theories). Once an appropriate number of counterterms are fixed in terms of physical observables, EFTs can be fully predictive within their range of validity and at a given order in the coupling/energy expansion (cf. e.g. the case of the SM EFT [24–26]). Nevertheless, renormalizable setups provide us with a larger degree of predictivity and in the following we will focus for simplicity on UV-complete, weakly coupled models.

4 Renormalizable approach to the $(g-2)_\mu$

In this section we discuss the case in which the new vector is embedded in a spontaneously broken extended gauge symmetry. Hence, $k_Q = 1$ in Eq. (11), so that the vector contribution to the $(g-2)_\mu$ turns out to be finite and predicted in terms of a renormalizable Lagrangian.

Before turning to the actual discussion of the gauge embeddings, we estimate the mass scale M_X required in order to explain the $(g-2)_\mu$ discrepancy, regardless of its UV gauge completion. Since existing bounds require $M_X \gg M_W$,[3] it is more appropriate to employ an $SU(2)_L \otimes U(1)_Y$ invariant language. To this purpose, we have classified in Appendix A all the possible X-quantum numbers such that the new vector can couple to SM fields via $d \leq 4$ operators (cf. Tables 2–3). Only a subset of these operators are relevant for the $(g-2)_\mu$, namely all those involving a lepton field, which are reported in Table 1. This Table summarizes most of our results. It shows all the possible new vector's quantum numbers, together with their EM components and the $d = 4$ operators involving X, a muon and a SM fermion field. Moreover, it contains, for each case, the sign of the contribution to Δa_μ in the approximation where the $SU(2)_L$ multiplet components have the same mass M_X,[4] and the value of M_X which is required in order to explain the experimental discrepancy, for the reference gauge coupling $g_X = 1$ (M_X scales linearly with g_X). Finally, in the last column, we provide a minimal gauge embedding of the massive vector into an extended gauge symmetry group. What we did not include in Table 1 are the actual bounds on M_X, which are instead discussed in detail in Sect. 4.2. In some cases a model-independent bound applies (namely without specifying the embedding), while in general the gauge embedding implies extra indirect constraints. As a matter of fact, we find that only the abelian extension can provide an explanation of the $(g-2)_\mu$ discrepancy, compatibly with the existing bounds.

4.1 Unitary gauge calculation

Let us consider the Lagrangian in Eq. (11) with $k_Q = 1$. The contribution to the muon anomalous magnetic moment (cf. the two diagrams displayed in Fig. 1) in the unitary gauge is known since long [27] (see also Refs. [2, 19]). At the leading order in m_μ/M_X and m_f/M_X it reads

$$\begin{aligned}\Delta a_\mu^{(a)} &= \frac{N_c Q_X}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[|g_V|^2\left(-\frac{5}{6}+\frac{m_f}{m_\mu}\right)+|g_A|^2\left(-\frac{5}{6}-\frac{m_f}{m_\mu}\right)\right] \\ &= \frac{N_c Q_X}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[-\frac{5}{12}\left(|g_L|^2+|g_R|^2\right)+\text{Re}\,(g_L g_R^*)\frac{m_f}{m_\mu}\right], \quad (15)\end{aligned}$$

$$\begin{aligned}\Delta a_\mu^{(b)} &= \frac{N_c Q_f}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[|g_V|^2\left(\frac{2}{3}-\frac{m_f}{m_\mu}\right)+|g_A|^2\left(\frac{2}{3}+\frac{m_f}{m_\mu}\right)\right] \\ &= \frac{N_c Q_f}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[\frac{1}{3}\left(|g_L|^2+|g_R|^2\right)-\text{Re}\,(g_L g_R^*)\frac{m_f}{m_\mu}\right], \quad (16)\end{aligned}$$

[3] The only exception is given by abelian gauge extensions (cf. end of Sect. 4.2.1). But in such a case X is a SM gauge singlet.

[4] The mass splitting between the electroweak components of an $SU(2)_L$ multiplet originates from a tree-level term and, for $M_X \gg M_W$, goes like $\Delta M_X \sim g'^2$ 10 GeV (1 TeV$/M_X$), where g' is a custodial breaking gauge coupling.

X^μ	$Q_{\rm EM}$	$\mathcal{O}_X^{g-2}$	sign(Δa_μ)	M_X[GeV]	Gauge embedding
$(1,1,0)$	0	$\bar{e}_R\gamma_\mu e_R X^\mu$, $\bar{\ell}_L\gamma_\mu \ell_L X^\mu$	$+/-$	180(220)	$U(1)'$
$(1,2,-\frac{3}{2})$	$-1,-2$	$\bar{e}_R\gamma_\mu \ell^c_L X^\mu$	$+$	750(900)	$SU(3)_L\otimes U(1)_X$
$(1,3,0)$	$1,0,-1$	$\bar{\ell}_L\gamma_\mu \ell_L X^\mu$	$+$	160(190)	$SU(2)_1\otimes SU(2)_2$
$(\overline{3},1,-\frac{2}{3})$	$-\frac{2}{3}$	$\bar{e}_R\gamma_\mu d_R X^\mu$, $\bar{\ell}_L\gamma_\mu q_L X^\mu$	$+/-$	2000(2400)	$SU(4)_C\otimes U(1)_R$
$(\overline{3},1,-\frac{5}{3})$	$-\frac{5}{3}$	$\bar{e}_R\gamma_\mu u_R X^\mu$	$+$	520(620)	$SU(4)_C\otimes U(1)_{R'}$
$(3,2,\frac{1}{6})$	$\frac{2}{3},-\frac{1}{3}$	$\bar{\ell}_L\gamma_\mu u^c_R X^\mu$	$-$	/	$SU(5)\otimes U(1)_Z$
$(3,2,-\frac{5}{6})$	$-\frac{1}{3},-\frac{4}{3}$	$\bar{e}_R\gamma_\mu q^c_L X^\mu$, $\bar{\ell}_L\gamma_\mu d^c_R X^\mu$	$+/-$	4400(5300)	$SU(5)$
$(\overline{3},3,-\frac{2}{3})$	$\frac{1}{3},-\frac{2}{3},-\frac{5}{3}$	$\bar{\ell}_L\gamma_\mu q_L X^\mu$	$+$	540(650)	$SO(9)\otimes U(1)_R$

Table 1: List of new Lorentz vectors coupling to SM fermions at the renormalizable level and contributing to the $g-2$. In the second column we provide the EM components of the $SU(2)_L$ multiplets, while $\mathcal{O}_X^{g-2}$ denotes the $d=4$ operator responsible for the $g-2$ (gauge and flavor indices are suppressed). Representations with $Y=0$ are understood to be real. For those cases where the contribution to the $(g-2)_\mu$ is non-negative we estimate in the fifth column the mass scale of the vector boson required in order to fit the discrepancy $\Delta a_\mu = (290\pm 90)\times 10^{-11}$ (the number in the bracket corresponds to the $+1\sigma$ value). For the estimate we take the gauge coupling $g_X=1$ and an universal mass M_X for all the components of the $SU(2)_L$ multiplets. The last column displays the minimal embedding of the massive vector into an extended gauge group.

where $Q_{X,f}$ denote the EM charges of X and f, while $N_c=3$ (1) for color triplets (singlets). Note that in the second step of Eqs. (15)–(16) we switched to the chiral basis couplings $g_L = g_V - g_A$ and $g_R = g_V + g_A$, which is a better language for $SU(2)_L\otimes U(1)_Y$ invariant interactions. The generalization in flavor space for a generic gauge theory is also straightforward. The interaction term involving X, a muon and a SM fermion mass eigenstate field f_i reads

$$\bar{\mu}\left(g_L U_L^{\mu i}\gamma_\alpha P_L + g_R U_R^{\mu i}\gamma_\alpha P_R\right) f_i X^\alpha + \text{h.c.}\,, \tag{17}$$

where $P_{L,R}=\frac{1}{2}(1\mp\gamma_5)$ are chiral projectors and $U_{L,R}$ are unitary matrices in flavor space which perform the rotation from the flavor to the mass basis. Consequently, Eqs. (15)–(16) are generalized into

$$\Delta a_\mu^{(a)} = \frac{N_c Q_X}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[-\frac{5}{12}\left(|g_L|^2+|g_R|^2\right) + \text{Re}\,(g_L g_R^*)\,\text{Re}\,\left(U_L^{\mu i}U_R^{*\mu i}\right)\frac{m_{f_i}}{m_\mu}\right]\,, \tag{18}$$

$$\Delta a_\mu^{(b)} = \frac{N_c Q_f}{4\pi^2}\frac{m_\mu^2}{M_X^2}\left[\frac{1}{3}\left(|g_L|^2+|g_R|^2\right) - \text{Re}\,(g_L g_R^*)\,\text{Re}\,\left(U_L^{\mu i}U_R^{*\mu i}\right)\frac{m_{f_i}}{m_\mu}\right]\,, \tag{19}$$

where in the first term of the square brackets we exploited the unitarity relation $(U_{L,R}U_{L,R}^\dagger)^{\mu\mu}=1$. On the other hand, the LR contribution in Eqs. (18)–(19) is weighted by the fermion mass m_{f_i} and, depending on the specific UV gauge completion, by a generally unknown unitary matrix element.

In reproducing the unitary gauge calculation we would like to mention a subtlety that one encounters when employing the unitary gauge at the loop level.[5] It is known that one should not shift momenta in more-than-logarithmically divergent integrals, otherwise spurious surface terms can change the final result by a finite amount (see e.g. Chapter 6.2 in [28]). This is a potential issue in the unitary gauge, since the degree of superficial divergence of the loop integrals gets worsened. Though the contribution to the $g-2$ must be finite in a renormalizable theory, one still needs to regularize the integrals in order not to meet the aforementioned issue. Indeed, we verified that the result of the calculation differs by a finite amount if one naively computes the integrals in $d = 4$ dimensions, instead of using dimensional regularization in $d = 4 - 2\epsilon$ and taking the $\epsilon \to 0$ limit at the very end.

4.2 New vectors' contributions, gauge embeddings and bounds

We proceed now by detailing the contribution of the new vectors in Table 1 to the $(g-2)_\mu$ by using Eqs. (18)–(19) and estimate in turn the value of M_X which is required in order to explain the discrepancy. Next, we discuss for each case a minimal gauge embedding of the new Lorentz vector. In order for the model to be phenomenologically viable, the SM fermions and Higgs boson must be properly embedded into the extended matter multiplets and the absence of gauge anomalies must be fulfilled. Regarding these last two points, we will not enter too much into details, but just refer to the existing literature when possible. For those cases where the SM matter embedding has not been discussed yet we will see that there exist independent arguments which are actually sufficient in order to exclude those possibilities as an explanation of the $(g-2)_\mu$. In particular, for any minimal viable realization we estimate indirect bounds from B and L number and flavor violating processes as well as limits from direct searches. To simplify the notation, we esplicitate the flavor structure only when needed. It is otherwise understood a unitary structure like that in Eq. (17), as the most general gauge interaction of the massive vector with the SM matter fields.

4.2.1 $(1, 1, 0)$

Sticking to a flavor diagonal Z', the interaction Lagrangian is

$$\mathcal{L}_{\text{int}}^{g-2} \supset g_{X_1} \overline{e}_R \gamma_\mu e_R X^\mu + g_{X_2} \overline{\ell}_L \gamma_\mu \ell_L X^\mu \supset g_{X_1} \overline{e}_R \gamma_\mu e_R X^\mu + g_{X_2} \overline{e}_L \gamma_\mu e_L X^\mu \,, \tag{20}$$

which yields

$$\Delta a_\mu = -\frac{1}{12\pi^2} \frac{m_\mu^2}{M_X^2} \left(g_{X_1}^2 + g_{X_2}^2 - 3 g_{X_1} g_{X_2} \right) \,. \tag{21}$$

The latter is positive for $\frac{1}{2}(3-\sqrt{5}) < g_{X_2}/g_{X_1} < \frac{1}{2}(3+\sqrt{5})$, while it reaches its maximal positive value

$$\Delta a_\mu^{\max} = \frac{g_{X_1}^2}{12\pi^2} \frac{m_\mu^2}{M_X^2} \frac{5}{4} \,, \tag{22}$$

for $g_{X_2}/g_{X_1} = 3/2$.[6] From Eq. (22) we find that the $(g-2)_\mu$ requires $M_X / g_{X_1} = 200$ GeV.

[5] Another option could be that of using a different gauge, like the 't Hooft-Feynman gauge.

[6] This option is prone to gauge anomaly cancellation constraints, since the couplings are chiral. However, anomalies can be fixed by coupling X to another fermionic sector. Alternatively, one can consider the anomaly free scenario $g_{X_1} = g_{X_2}$. In such a case, the required vector boson mass is $M_X / g_{X_1} = 180$ GeV.

The gauge embedding corresponds to that of an extra $U(1)'$ factor and the lower bounds on M_X are quite model dependent. For instance, in the case of a sequential SM Z', ATLAS [29] and CMS [30] set the bound respectively to $M_{Z'} > 3.4$ TeV and $M_{Z'} > 3.2$ TeV by looking into di-lepton channels. On the other hand, even if the Z' couples only to muons (as minimally required by the muon $g-2$), neutrino trident production $\nu_\mu N \to \nu_\mu N \mu^+\mu^-$ from CCFR data [31] rules out the explanation of the $(g-2)_\mu$ anomaly for masses $M_{Z'} \gtrsim 400$ MeV [32], while the available low-mass range can be covered at future neutrino beam facilities.

Without requiring additional exotic fermions contributing to the $(g-2)_\mu$, there are two other options leading to a viable Z' explanation of the $(g-2)_\mu$. The first one is a dark photon or Z without direct couplings to the SM fields, which can still contribute to the $(g-2)_\mu$ via a gauge kinetic mixing to the EM current [33–35]. As shown in Ref. [35], the explanation in terms of a light gauge boson of $\mathcal{O}(100)$ MeV requires however a sizeable invisible decay channel of the Z'. Another possibility, recently discussed in Refs. [36, 37], is that of a flavor off-diagonal coupling of the Z' to the μ and τ sector. This can explain the $(g-2)_\mu$ for a Z' heavier than the τ lepton, compatibly with all the existing bounds.

4.2.2 $(1, 2, -\frac{3}{2})$

Let us consider the interaction Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{int}}^{g-2} &\supset g_X \overline{e}_R \gamma_\mu \ell_L^c X^\mu + \text{h.c.} = g_X \left[\overline{e}_R \gamma_\mu \nu_L^c X_{-1}^\mu + \overline{e}_R \gamma_\mu e_L^c X_{-2}^\mu\right] + \text{h.c.} \\ &= g_X \left[\overline{e}_R \gamma_\mu \nu_L^c X_{-1}^\mu + \left(\tfrac{1}{2} \underbrace{\overline{e} \gamma_\mu C \overline{e}^T}_{=0} X_{-2}^\mu + \tfrac{1}{2} \overline{e} \gamma_\mu \gamma_5 C \overline{e}^T X_{-2}^\mu\right)\right] + \text{h.c.}\,,\end{aligned} \tag{23}$$

where in the last step we have emphasized the fact that the vector current associated to the doubly-charged component of X is zero by symmetry reasons.[7] Note, also, that the Feynman rule of X_{-2} features an extra 2 symmetry factor in the $\mu\mu X$ vertex (and hence a factor 4 in the $g-2$ amplitude). At the end the final contribution to the $(g-2)_\mu$ is found to be

$$\Delta a_\mu = \frac{23}{16\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2\,, \tag{24}$$

and in order to reproduce the $(g-2)_\mu$ we need $M_X/g_X = 750$ GeV.

The gauge embedding in this case is minimally realized via the so-called 331 models, which are based on the extended gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [38]. The SM hypercharge is embedded via the relation

$$Y = \xi T_L^8 + X\,, \tag{25}$$

where T_L^8 is a Cartan generator of the $SU(3)_L$ algebra (normalized as $\mathrm{Tr}\, T_L^a T_L^b = \frac{1}{2}\delta^{ab}$) and the parameter ξ defines a class of different models (see e.g. [39]), while the X-charge assignment of the matter fields defines the embedding of the SM fermions into the extended matter multiplets. In particular, in order to obtain $(1, 2, -\frac{3}{2})$ as a would-be goldstone boson (WBG) one needs $\xi = \pm\sqrt{3}$. On top of that, the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ breaking also delivers a Z'.

[7] In fact, by using the anticommuting properties of fermion fields, $C\gamma^{\mu T} C^{-1} = -\gamma^\mu$ and $C^T = -C$ one gets: $\overline{e}\gamma_\mu C \overline{e}^T = (\overline{e}\gamma_\mu C \overline{e}^T)^T = -\overline{e} C^T \gamma_\mu^T \overline{e}^T = -\overline{e}\gamma_\mu C \overline{e}^T$.

Ref. [40] studied the interplay between the $(g-2)_\mu$ and the electroweak and collider constraints in different classes of 331 models and found that no renormalizable extension can explain the $(g-2)_\mu$, mainly due to lower bounds on the Z' mass which translate into lower bounds on the singly and doubly charged components of $(1,2,-\frac{3}{2})$ within the specific models.

4.2.3 $(1,3,0)$

In this case we use a matrix representation for the (real) electroweak triplet

$$\mathbf{X}^\mu = \frac{\sigma^i X^{i\mu}}{\sqrt{2}} = \begin{pmatrix} \frac{X_0^\mu}{\sqrt{2}} & X_{+1}^\mu \\ X_{-1}^\mu & -\frac{X_0^\mu}{\sqrt{2}} \end{pmatrix} , \tag{26}$$

and the relevant Lagrangian for the $(g-2)_\mu$ is

$$\mathcal{L}_{\rm int}^{g-2} \supset g_X \overline{\ell}_L \gamma_\mu \mathbf{X}^\mu \ell_L \supset -\frac{g_X}{\sqrt{2}} \overline{e}_L \gamma_\mu e_L X_0^\mu + g_X \left(\overline{e}_L \gamma_\mu \nu_L X_{-1}^\mu + \text{h.c.} \right) . \tag{27}$$

The contribution to Δa_μ is

$$\Delta a_\mu = \frac{1}{16\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2 , \tag{28}$$

from which we get that in order to reproduce the $(g-2)_\mu$ we need $M_X/g_X = 160$ GeV.

A minimal gauge extensions delivering $(1,3,0)$ as a WBG is given by $SU(2)_1 \otimes SU(2)_2$, spontaneously broken to the diagonal subgroup, which is identified with $SU(2)_L$. Different variant models depend on the SM fermions' embedding. Let us mention, for instance, the "un-unified" model where left-handed quarks and leptons are respectively assigned to $SU(2)_1$ and $SU(2)_2$ [41, 42], and the "non-universal" model in which the third generation left-handed fermions undergo a different $SU(2)$ interaction from those of the first two generations [43]. Due to the symmetry breaking pattern the masses of the W' and Z' contained in the $(1,3,0)$ are quite degenerate and their mixing with the W and Z leads to strong constraints from precision electroweak measurements. In fact, a global analysis including Z-pole observables, W properties, τ lifetime, $\nu N(e)$-scattering and atomic parity violation sets the bound at the level of $M_{W'} \sim M_{Z'} \gtrsim 2.5$ TeV [44]. On the other hand, the new charged vector bosons can be pair-produced and leave a signature of leptons and missing energy. By recasting LHC slepton searches [45], Ref. [18] sets the lower bound $M_{W'} \gtrsim 400$ GeV, which holds irrespectively of the UV completion. This clearly rules out the possible explanation of the $(g-2)_\mu$ discrepancy.

4.2.4 $(\overline{3},1,-\frac{2}{3})$

The interaction Lagrangian can be written as

$$\mathcal{L}_{\rm int}^{g-2} \supset g_X \left(\overline{e}_R \gamma_\mu d_R X^\mu + \overline{\ell}_L \gamma_\mu q_L X^\mu \right) + \text{h.c.} \supset g_X \left(\overline{e}_{R\mu} U_R^{\mu i} \gamma_\mu d_{Ri} X^\mu + \overline{e}_{L\mu} U_L^{\mu i} \gamma_\mu d_{Li} X^\mu \right) + \text{h.c.} , \tag{29}$$

where $U_{L,R}$ are unitary mixing matrices. The contribution to the $(g-2)_\mu$ is then

$$\Delta a_\mu = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2 \left(1 - \text{Re} \left(U_L^{\mu i} U_R^{*\mu i} \right) \frac{m_{d_i}}{m_\mu} \right) . \tag{30}$$

In order to maximize the contribution, we assume maximal mixing in the bottom direction with $\mathrm{Re}\left(U_L^{\mu i}U_R^{*\mu i}\right) = -1$, thus inferring $M_X/g_X = 2.0$ TeV in order to explain the $(g-2)_\mu$ discrepancy.

The minimal UV completion of the $(\overline{3}, 1, -\frac{2}{3})$ vector leptoquark is given by the quark-lepton unification model based on the gauge group $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$ (see e.g. [46]), which is a particular case of the more general Pati-Salam group [47]. The SM hypercharge is embedded via the relation

$$Y = \frac{\sqrt{6}}{3} T_C^{15} + R\,, \tag{31}$$

where T_C^{15} is a properly normalized Cartan generator of $SU(4)_C$ algebra ($\mathrm{Tr}\, T_C^a T_C^b = \frac{1}{2}\delta^{ab}$). The R-charge assignment of the matter fields defines the embedding of the SM fermions into the extended matter multiplets. On top of $(\overline{3}, 1, -\frac{2}{3})$, the $SU(4)_C \otimes SU(2)_L \otimes U(1)_R \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ breaking also delivers a Z' as a WBG.

The vector leptoquark $(\overline{3}, 1, -\frac{2}{3})$ contributes to the rare decay $K_L^0 \to e^\mp \mu^\mp$, which for $\mathcal{O}(1)$ couplings yields the bound $M_X \gtrsim 10^3$ TeV (see e.g. [48, 49]). Such a strong constraint can be in principle evaded if one takes into account the freedom in the flavor mixing between quarks and leptons, due to the unknown unitarity matrices $U_{L,R}$ in Eq. (29). In such a case, a full set of observables from rare K and B meson decays must be taken into account and, by combining the strongest constraints, Refs. [50, 51] find $M_X \gtrsim 38$ TeV, regardless of flavor mixing. Remarkably, a numerical scan of the multi-dimensional parameter space reveals the existence of viable configurations with masses as low as $M_X \sim 12$ TeV [52], which is however still too high for the explanation of the $(g-2)_\mu$.

4.2.5 $(\overline{3}, 1, -\frac{5}{3})$

From the interaction Lagrangian

$$\mathcal{L}_{\text{int}}^{g-2} \supset g_X \overline{e}_R \gamma_\mu u_R X^\mu + \text{h.c.}\,, \tag{32}$$

the contribution to Δa_μ is found to be

$$\Delta a_\mu = \frac{11}{16\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2\,. \tag{33}$$

The mass scale required to fit the $(g-2)_\mu$ is $M_X/g_X = 520$ GeV.

The gauge extension of this case is analogous to the previous one, and is given by the $SU(4)_C \otimes SU(2)_L \otimes U(1)_{R'}$ group, whose breaking also delivers an extra Z' as a WBG. The corresponding embedding of the SM hypercharge is

$$Y = \frac{5}{2} \frac{\sqrt{6}}{3} T_C^{15} + R'\,. \tag{34}$$

The R'-charges of the matter fields define the embedding of the SM fermions into the extended matter multiplets. The latter differs substantially from the standard Pati-Salam embedding and we did not attempt to build a realistic fermionic sector. However, even without discussing that, such a light M_X (as required by the $(g-2)_\mu$) is ruled out by collider searches.

In order to show that let us make explicit the unitary structure of the leptoquark interactions in flavor space

$$g_X U_R^{ij} \overline{e}_{iR} \gamma_\mu u_{jR} X^\mu\,, \tag{35}$$

where the (a priori unknown) unitary matrix U_R controls the branching ratios of $X \to e_i u_j$. In particular, we have

$$\mathcal{B}(X \to ej) = \frac{\sum_{j=u,c} \left|U_R^{ej}\right|^2}{\sum_{i=e,\mu,\tau} \sum_{j=u,c,t} \left|U_R^{ij}\right|^2} = \frac{1 - |U_R^{et}|^2}{3} \,, \tag{36}$$

$$\mathcal{B}(X \to \mu j) = \frac{\sum_{j=u,c} \left|U_R^{\mu j}\right|^2}{\sum_{i=e,\mu,\tau} \sum_{j=u,c,t} \left|U_R^{ij}\right|^2} = \frac{1 - \left|U_R^{\mu t}\right|^2}{3} \,. \tag{37}$$

On the other hand, the pair-production cross section of X is unambiguously fixed by QCD and we can use the CMS searches in Ref. [53] in order to constrain the combined $X \to ej$ and $X \to \mu j$ channels. Note that the elements U_R^{ej} and $U_R^{\mu j}$ are still related by unitarity, and even in the worse case scenario where the top is maximally mixed with the first two generation leptons (thus leading to a potential reduction of the branching ratios in Eqs. (36)–(37)), we can parametrize the mixing matrix elements as $U_R^{ej} = \sin\phi$ and $U_R^{\mu j} = \cos\phi$. The most conservative bound is obtained by simultaneously minimizing the two branching ratios, since ej and μj searches lead to similar bounds. This is obtained by taking $\phi = \pi/4$, which corresponds to a $\mathcal{B}$ of $1/6$ in both the channels. By simply rescaling the cross sections in Figs. 13 and 14 of Ref. [53] by a $(1/6)^2$ factor we obtain $M_X \gtrsim 1$ TeV, which is sufficient in order to exclude the explanation of the $(g-2)_\mu$ in terms of $(\overline{3}, 1, -\frac{5}{3})$.

4.2.6 $(3, 2, \frac{1}{6})$

Given the interaction Lagrangian

$$\mathcal{L}_{\text{int}}^{g-2} \supset g_X \overline{\ell}_L \gamma_\mu u_R^c X^\mu + \text{h.c.} \supset g_X \overline{e}_L \gamma_\mu u_R^c X_{-1/3}^\mu + \text{h.c.} \,, \tag{38}$$

the contribution to Δa_μ is

$$\Delta a_\mu = -\frac{1}{16\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2 \,, \tag{39}$$

which, being negative, cannot explain the $(g-2)_\mu$.

For completeness, we mention that this case corresponds to the "flipped" SU(5) embedding of the SM hypercharge [54, 55]. Moreover, the breaking also delivers an extra Z' as a WBG.

4.2.7 $(3, 2, -\frac{5}{6})$

The interaction Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{\text{int}}^{g-2} &\supset g_X \left(\overline{e}_R \gamma_\mu q_L^c X^\mu + \overline{\ell}_L \gamma_\mu d_R^c X^\mu \right) + \text{h.c.} \\ &\supset g_X \left(\overline{e}_{R\mu} \tilde{U}_R^{\mu i} \gamma_\mu u_{Li}^c X_{-1/3}^\mu + \overline{e}_{R\mu} U_R^{\mu i} \gamma_\mu d_{Li}^c X_{-4/3}^\mu + \overline{e}_{L\mu} U_L^{\mu i} \gamma_\mu d_{Ri}^c X_{-4/3}^\mu \right) + \text{h.c.} \,, \end{aligned} \tag{40}$$

where $\tilde{U}_R$ and $U_{L,R}$ are unitary mixing matrices. The contribution to Δa_μ is found to be

$$\Delta a_\mu = \frac{5}{16\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2 \left(3 - 4 \,\text{Re}\, \left(U_L^{\mu i} U_R^{*\mu i} \right) \frac{m_{d_i}}{m_\mu} \right) \,. \tag{41}$$

In order to maximize the contribution, we assume maximal mixing in the bottom direction with Re $\left(U_L^{\mu i} U_R^{*\mu i}\right) = -1$, and thus we get $M_X/g_X = 4.4$ TeV in order to explain the $(g-2)_\mu$ discrepancy.

The UV completion of this vector leptoquark is the standard $SU(5)$ [56], which clearly rules out the interpretation of the $(g-2)_\mu$, since $M_X \gtrsim 10^{15}$ GeV from proton decay and unification constraints.

4.2.8 $(\overline{3}, 3, -\frac{2}{3})$

By using the following electroweak triplet matrix representation

$$\mathbf{X}^\mu = \frac{\sigma^i X^{i\mu}}{\sqrt{2}} = \begin{pmatrix} \frac{X^\mu_{-2/3}}{\sqrt{2}} & X^\mu_{+1/3} \\ X^\mu_{-5/3} & -\frac{X^\mu_{-2/3}}{\sqrt{2}} \end{pmatrix} , \tag{42}$$

we can write the interaction Lagrangian as

$$\mathcal{L}_{\rm int}^{g-2} \supset g_X \overline{\ell}_L \gamma_\mu \mathbf{X}^\mu q_L + \text{h.c.} \supset -\frac{g_X}{\sqrt{2}} \overline{e}_L \gamma_\mu d_L X^\mu_{-2/3} + g_X \overline{e}_L \gamma_\mu u_L X^\mu_{-5/3} + \text{h.c.} \, . \tag{43}$$

This leads to

$$\Delta a_\mu = \frac{3}{4\pi^2} \frac{m_\mu^2}{M_X^2} g_X^2 \, , \tag{44}$$

which implies $M_X/g_X = 540$ GeV for the explanation of the $(g-2)_\mu$ discrepancy.

On top of possible collider searches which we do not discuss, the main no-go here is the gauge embedding which requires the $SU(3)_C$ and $SU(2)_L$ SM gauge factor to get unified below the TeV scale, which is clearly ruled out.

For completeness, we provide a symmetry breaking pattern delivering $(\overline{3}, 3, -\frac{2}{3})$ as a WBG. The minimal option we were able to find is $SO(9) \otimes U(1)_R \to SU(4)_C \otimes SU(2)_L \otimes U(1)_R \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Here, the branching rule of the adjoint under $SO(9) \to SU(4)_C \otimes SU(2)_L$ is given by $36 \to (1,3) \oplus (6,3) \oplus (15,1)$ [57]. Next, under $SU(4)_C \otimes SU(2)_L \otimes U(1)_R \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, $(6,3,0) \to (3,3,\frac{2}{3}) \oplus (\overline{3},3,-\frac{2}{3})$, provided the embedding of the SM hypercharge is $Y = \frac{2\sqrt{6}}{3} T_C^{15} + R$. On the other hand, the embedding of the SM fermions is non-trivial and we did not attempt to build a realistic model.

5 Discussion and conclusions

The increase of the degree of divergence of loop diagrams in presence of non-gauge massive vectors is something well-known. A typical example is given by meson mixing amplitudes for which the box diagrams involving massive vectors, with propagators as in Eq. (5), are quadratically divergent (see e.g. [58, 59])

$$\Delta m_M^{ij} \propto \Lambda^2 \sum_{f,f'} U^{if} U^{*jf} U^{if'} U^{*jf'} \, . \tag{45}$$

Here, M denotes a K, D or B meson, ij are the meson constituent quarks and ff' the fermions exchanged in the loop. In a gauge theory the U-matrices are unitary and a GIM-like mechanism ensures the cancellation of the quadratic divergence, as it should in a renormalizable theory. Yet another example is given by the divergent contributions to electroweak precision observables from composite vectors (see e.g. [60–64]). In a similar way, we have seen that the triple vector boson vertex in diagram (a) of Fig. 1 is the origin of the logarithmic divergence of the $g-2$. This is also to be expected, since renormalizability crucially hinges on the exact values of the non-abelian vertices.

In this paper we have classified all the possible quantum numbers of a new massive vector which can couple to SM fields via $d \leq 4$ operators (cf. Tables 2–3). Only a subset of these irreps can contribute to the $g-2$, and for each of them we provided the embedding of the massive vector into a spontaneously broken gauge theory (cf. Table 1). While some gauge extensions are of course well-known, those concerning $(\overline{3}, 1, -\frac{5}{3})$ and $(\overline{3}, 3, -\frac{2}{3})$ are to our knowledge new. The maybe less obvious result of this paper is that after embedding the massive vector into an extended gauge symmetry, such that the $g-2$ can be unambiguously computed in terms of a renormalizable Lagrangian, renormalizability highly constrains the interactions of the vector field. In fact, a combination of direct and indirect bounds, as well as unification constraints, rules out the possible explanation of the muon $g-2$ in terms of new massive vectors, with the only notable exception of an abelian gauge extension. The latter, indeed, is less constrained because the extra gauge group is factorized with respect to the SM gauge group and the couplings to SM fields are highly model dependent.

It should be also stressed that the starting hypothesis of a 1-particle vector extension of the SM is often violated in the renormalizable case, since new sectors of the theory are often required by the consistency of the symmetry breaking pattern (e.g. scalar fields breaking the extended symmetry, extra WGBs and new fermions fitting the extended matter multiplets) and they cannot be arbitrary decoupled from the new vector mass scale. In principle, the inclusion of these extra fields can provide extra contribution for explaining the $(g-2)_\mu$. This, however, is model dependent and goes beyond the original question.

Finally, we would like to comment on a couple of other phenomenologically relevant contexts where similar observations apply as in the $g-2$ case. The first one is that of B-meson decay anomalies. New massive vectors have been recently proposed for addressing some 3σ level discrepancies in semileptonic B-meson decays [3–5]. Aside from abelian gauge extensions (see e.g. [65–70]) there are three non-trivial irreps which are well-suited for addressing B-meson anomalies if they couple to SM fermions exclusively via left-handed currents: $(1, 3, 0)$ [71–74], $(\overline{3}, 1, -\frac{2}{3})$ [23, 75] and $(\overline{3}, 3, -\frac{2}{3})$ [76, 77]. In these examples the issue of renormalizability was not central, being all the main experimental anomalies to be explained at tree level (see however Ref. [23] for a discussion of divergent loop observables). Nevertheless, if one requires these non-abelian massive vectors to arise from a spontaneously broken extended gauge symmetry new extra constraints must be fulfilled. We already discussed a minimal gauge embedding for each of these three vector irreps in Sect. 4.2. As far as regards $(1, 3, 0)$, if it couples universally to the three SM families, the unitarity of the gauge interactions forces the neutral currents to be diagonal in flavor space and the charged currents to be aligned to the SM, thus lacking of the required amount of flavor violation for $b \to s$ and $b \to c$ transitions. As pointed out in Refs. [73, 74], a viable UV gauge completion of $(1, 3, 0)$ for the explanation of the the B-anomalies requires universal gauge couplings and an extra source of flavor violation, e.g. from the mixing of the SM quarks with new vector-like fermions. Similarly, in the case of $(\overline{3}, 1, -\frac{2}{3})$ the unitary structure of the leptoquark interactions with the SM fermions is such that a bunch of rare processes from rare K and B meson decays cannot be simply set to zero by switching-off right-handed currents. As discussed in Sect. 4.2.4, the mass of the new vector is bounded to lie in the multi-tens of TeV region and hence too high in order to explain all the B anomalies. Finally, the case of a light $(\overline{3}, 3, -\frac{2}{3})$ is also trivially excluded, since if it were to come from a gauge theory the strong and electroweak couplings would have to be unified at the TeV scale.

Massive vectors mediators have been also recently invoked for the explanation of the LHC diphoton excess (see e.g. [78, 79]). In such a case both the production of the scalar resonance via gluon fusion and its decay into two photons is obtained via a loop of massive vectors featuring triple and quartic vector boson vertices, which lead in general to divergent contributions. On the other hand, by sticking to a finite result for the loop functions in order to fit the cross-section signal one is implicitly assuming that the vector boson has a gauge origin and, as we saw in the previous examples, it is non-trivial to satisfy all the relevant bounds in presence of a gauge vector mediator at the TeV scale.

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A 1-particle vector extensions of the SM

In this Appendix we provide the classification of all the possible gauge quantum numbers of a Lorentz vector, X^μ, which can couple to SM fields at the renormalizable level. We start by collecting in Table 2 those cases where the new vector couples to SM fermions. In Table 3 we classify instead $d \leq 4$ operators involving X^μ and SM bosons (either scalar or vector).

Note that some of the operators collected in Table 3 can potentially yield extra non-standard contributions to the $g-2$. This happens for the operator $W_{\mu\nu} D^\mu X^\nu$, which only exists when X transforms like $(1, 3, 0)$, or for some operators involving the ϵ-tensor. It can be shown, however, that the former operator does not arise in renormalizable setups. To this end, let us consider the gauge embedding of the $SU(2)_L$ factor in terms of $SU(2)_1 \otimes SU(2)_2$ discussed in Sect. 4.2.3: the only possible source of such operator is the kinetic term of the two field strengths

$$-\frac{1}{4} W_1^{a,\mu\nu} W^a_{1,\mu\nu} - \frac{1}{4} W_2^{a,\mu\nu} W^a_{2,\mu\nu} \,, \tag{46}$$

which upon an orthogonal transformation in terms of mass eigenstates, namely a massless triplet W and a massive one X (we neglect electroweak symmetry breaking here), leads to

$$-\frac{1}{4} W^{a,\mu\nu} W^a_{\mu\nu} - \frac{1}{4} X^{a,\mu\nu} X^a_{\mu\nu} \,, \tag{47}$$

without any W-X mixed term. Similarly, among the operators obtained via an $\epsilon_{\mu\nu\rho\sigma}$ contraction, those arising from renormalizable theories are always total derivatives, and hence do not contribute to the $g-2$ in perturbation theory.

X^μ	$Q_{\rm EM}$	$\mathcal{O}_X^{d=4}$
$(1,1,0)$	0	$\overline{e}_R\gamma_\mu e_R X^\mu$, $\overline{\ell}_L\gamma_\mu\ell_L X^\mu$, $\overline{u}_R\gamma_\mu u_R X^\mu$, $\overline{d}_R\gamma_\mu d_R X^\mu$, $\overline{q}_L\gamma_\mu q_L X^\mu$
$(1,1,1)$	1	$\overline{u}_R\gamma_\mu d_R X^\mu$
$(1,2,-\frac{3}{2})$	$-1,-2$	$\overline{\ell}_L\gamma_\mu (e_R)^c X^\mu$
$(1,3,0)$	$1,0,-1$	$\overline{\ell}_L\gamma_\mu\ell_L X^\mu$, $\overline{q}_L\gamma_\mu q_L X^\mu$
$(\overline{3},1,-\frac{2}{3})$	$-\frac{2}{3}$	$\overline{e}_R\gamma_\mu d_R X^\mu$, $\overline{\ell}_L\gamma_\mu q_L X^\mu$
$(\overline{3},1,-\frac{5}{3})$	$-\frac{5}{3}$	$\overline{e}_R\gamma_\mu u_R X^\mu$
$(3,2,\frac{1}{6})$	$\frac{2}{3},-\frac{1}{3}$	$\overline{\ell}_L\gamma_\mu (u_R)^c X^\mu$, $\overline{q}_L^c\gamma_\mu d_R X^\mu$
$(3,2,-\frac{5}{6})$	$-\frac{1}{3},-\frac{4}{3}$	$\overline{e}_R\gamma_\mu q_L^c X^\mu$, $\overline{\ell}_L\gamma_\mu d_R^c X^\mu$, $\overline{q}_L^c\gamma_\mu u_R X^\mu$
$(\overline{3},3,-\frac{2}{3})$	$\frac{1}{3},-\frac{2}{3},-\frac{5}{3}$	$\overline{\ell}_L\gamma_\mu q_L X^\mu$
$(\overline{6},2,\frac{1}{6})$	$\frac{2}{3},-\frac{1}{3}$	$\overline{q}_L^c\gamma_\mu d_R X^\mu$
$(\overline{6},2,-\frac{5}{6})$	$-\frac{1}{3},-\frac{4}{3}$	$\overline{q}_L^c\gamma_\mu u_R X^\mu$
$(8,1,0)$	0	$\overline{u}_R\gamma_\mu u_R X^\mu$, $\overline{d}_R\gamma_\mu d_R X^\mu$, $\overline{q}_L\gamma_\mu q_L X^\mu$
$(8,1,1)$	1	$\overline{u}_R\gamma_\mu d_R X^\mu$
$(8,3,0)$	$1,0,-1$	$\overline{q}_L\gamma_\mu q_L X^\mu$

Table 2: List of new Lorentz vectors with $d=4$ coupling to SM fermions. The EM charges of the particles in the multiplet and the relevant $d=4$ operators are displayed (gauge and flavor indices are understood).

$\mathcal{O}_X$	$\dim(\mathcal{O}_X)$	X^μ
$HD_\mu X^\mu$	3	$(1,2,\frac{1}{2})$
$H^\dagger D_\mu X^\mu$	3	$(1,2,-\frac{1}{2})$
$HHD_\mu X^\mu$	4	$(1,3,-1)$
$HH^\dagger D_\mu X^\mu$	4	$(1,1\oplus 3,0)$
$HHX_\mu X^\mu$	4	$(R,2k,-\frac{1}{2})$
$HH^\dagger X_\mu X^\mu$	4	$(R,2k,0)$
$HH^\dagger X_\mu X^{\dagger\mu}$	4	(C,n,Y)
$D_\mu X_\nu^\dagger D^\mu X^\nu$	4	(C,n,Y)
$D_\mu X_\nu^\dagger D^\nu X^\mu$	4	(C,n,Y)
$G_{\mu\nu} D^\mu X^\nu$	4	$(8,1,0)$
$W_{\mu\nu} D^\mu X^\nu$	4	$(1,3,0)$
$B_{\mu\nu} \partial^\mu X^\nu$	4	$(1,1,0)$
$G_{\mu\nu} X^\mu X^{\dagger\nu}$	4	$(C_{\neq 1},n,Y)$
$W_{\mu\nu} X^\mu X^{\dagger\nu}$	4	$(C,n_{\neq 1},Y)$
$B_{\mu\nu} X^\mu X^{\dagger\nu}$	4	(C,n,Y)
$D_\mu X_\nu X^\mu X^\nu$	4	$(C,2k+1,0)$
$D_\mu X_\nu X^\mu X^{\dagger\nu}$	4	$(R,2k+1,0)$
$\epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} D^\rho X^\sigma$	4	$(8,1,0)$
$\epsilon_{\mu\nu\rho\sigma} W^{\mu\nu} D^\rho X^\sigma$	4	$(1,3,0)$
$\epsilon_{\mu\nu\rho\sigma} B^{\mu\nu} \partial^\rho X^\sigma$	4	$(1,1,0)$
$\epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} X^\rho X^{\dagger\sigma}$	4	$(C_{\neq 1},n,Y)$
$\epsilon_{\mu\nu\rho\sigma} W^{\mu\nu} X^\rho X^{\dagger\sigma}$	4	$(C,n_{\neq 1},Y)$
$\epsilon_{\mu\nu\rho\sigma} B^{\mu\nu} X^\rho X^{\dagger\sigma}$	4	(C,n,Y)

Table 3: New vectors X^μ which can couple to H or SM gauge bosons at the renormalizable level. (C,n,Y) denote generic quantum numbers under the SM gauge group. R stands for a real $SU(3)_C$ representation (i.e. $R = 1, 8, 27, \ldots$), while $2k$ $(2k+1)$ for an even (odd) $SU(2)_L$ representation. The subscript "$\neq 1$" means that the trivial representation is excluded.

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